

# Fermions Tunneling from Charged Accelerating and Rotating Black Holes with NUT Parameter

M. Sharif <sup>\*</sup> and Wajiha Javed <sup>†</sup>

Department of Mathematics, University of the Punjab,  
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

## Abstract

This paper is devoted to the study of Hawking radiation as a tunneling of charged fermions through event horizons of a pair of charged accelerating and rotating black holes with NUT parameter. We evaluate tunneling probabilities of outgoing charged particles by using the semiclassical WKB approximation to the general covariant Dirac equation. The Hawking temperature corresponding to this pair of black holes is also investigated. For the zero NUT parameter, we find results consistent with those already available in the literature.

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## 1 Introduction

*Hawking radiation* [1] is a quantum mechanical process which can be described as follows: Firstly, a pair of positive and negative energy particles is created near the horizon. The negative energy particle falls inside the horizon, while the positive energy particle tunnels outside the horizon and rises in as Hawking radiation for the outside observer. Secondly, the pair

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<sup>\*</sup>msharif.math@pu.edu.pk

<sup>†</sup>wajihajaved84@yahoo.com

of particles is created just inside the horizon. The positive energy particles have ability to cross the energy barrier and tunnels to infinity, however the negative energy partner remains behind. This reduces the mass of black hole (BH) by losing energy in the form of Hawking radiation. The process in which particles have finite probability to cross the event horizon (that cannot be possible for classical particles) is called *quantum tunneling*. In this process, we evaluate tunneling probabilities that depend on the imaginary part of the action for emitted particles through the horizon.

There are two methods to find the imaginary part of the action for the emitted particles. The first method was developed by Parikh and Wilczek [2] named as the *null geodesic method* by following the Kraus and Wilczek [3] technique. The second one is called *Hamilton–Jacobi method* which was first used by Angheben *et al* [4] for tunneling phenomenon to extend the complex path analysis of Srinivasan *et al* [5, 6]. The tunneling probability  $\Gamma$  for the emitted particle can be written as

$$\Gamma \propto \exp[-2\text{Im}I], \quad (1.1)$$

where  $I$  is the classical action ( $\hbar = 1$ ). An approximation, called *WKB approximation* was used by Wentzel, Kramers and Brillouin (WKB) [7] to develop a relation between classical and quantum theory. This is used to solve the Schrödinger equation ( $\hbar = 0$  leads to Hamilton–Jacobi equation in classical theory) in quantum mechanics. In both the above methods, the WKB approximation is used.

The study of Hawking radiation as a tunneling phenomenon from various BHs has attracted many people. Page [8] calculated the emission rates from an uncharged, nonrotating BH for massless particles of spin  $\frac{1}{2}$ , 1 and 2 by using the perturbation formalism. The same author [9] found numerical calculations of the emission rates of massless particles from a rotating BH. Kerner and Mann [10] extended the calculations of the tunneling process for the spin  $\frac{1}{2}$  particles emission by using the WKB approximation to the Dirac equation and found the tunneling probability from nonrotating BHs. Also, fermions tunneling is applied to a general nonrotating BH and recovered the corresponding Hawking temperature. The same authors [11] also discussed the tunneling of charged spin  $\frac{1}{2}$  fermions from a Kerr–Newman BH and obtained the Hawking temperature.

In recent papers, the tunneling probabilities of incoming and outgoing scalar and charged/uncharged fermions from accelerating and rotating BHs

have been investigated [12]. Also, the thermodynamical properties of accelerating and rotating BHs with Newman–Unti–Tamburino (NUT) parameter have been studied [13]. Recently, we have also explored some work [14] about the tunneling phenomenon for different BHs by using the above mentioned methods. In this paper, we extend this tunneling phenomenon of charged fermions from a pair of charged accelerating and rotating BHs with NUT parameter by using the procedure [10].

The paper is outlined as follows. In the next section, we briefly review Hamilton–Jacobi ansatz. Section 3 is devoted to explain the basic equations for a pair of accelerating and rotating BHs with NUT parameter having electric and magnetic charges. In section 4, we deal with the solution of the Dirac equation of charged particles in the background of these BHs and evaluate the tunneling probabilities as well as the corresponding temperature across the horizons. Section 5 provides an exact form of the action for the massless and massive particles. Finally, we summarize the results in the last section.

## 2 Hamilton–Jacobi Ansatz: An Overview

In this section, we briefly review Hawking radiation as tunneling through Hamilton–Jacobi ansatz [3, 4, 10]. The metric for a general (non-extremal) BH is

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + C(r)h_{ij}dx^i dx^j. \quad (2.1)$$

Assume a scalar particle moving in this BH. Since we are interested only in the leading terms within the semiclassical approximation, we neglect the effects of the particle self-gravitation. Thus the quantity which satisfies the relativistic Hamilton–Jacobi equation is the classical action  $I$

$$g^{\mu\nu}\partial_\mu I \partial_\nu I + m^2 = 0 \quad (2.2)$$

which implies that

$$-\frac{(\partial_t I)^2}{f(r)} + g(r)(\partial_r I)^2 + \frac{h^{ij}}{C(r)}\partial_i I \partial_j I + m^2 = 0. \quad (2.3)$$

Due to the symmetries of the metric, there exists a solution of the form

$$I = -Et + W(r) + J(x^i), \quad (2.4)$$

where  $\partial_t I = -E$ ,  $\partial_r I = W'(r)$ ,  $\partial_i I = J_i$  and the  $J_i$ 's are constants.  $E$  corresponds to energy as  $\partial_t$  is the timelike Killing vector. Solving for  $W(r)$ , it follows that

$$W_{\pm}(r) = \pm \int \frac{dr}{\sqrt{f(r)g(r)}} \sqrt{E^2 - f(r) \left( m^2 + \frac{h^{ij} J_i J_j}{C(r)} \right)}. \quad (2.5)$$

Here,  $W_+$  and  $W_-$  correspond to scalar particles moving away (outgoing) and moving towards the BH (incoming), respectively. Notice that the imaginary part of the action can only be due to the pole at the horizon.

The probabilities of crossing the horizon each way are

$$\text{Prob[out]} \propto \exp\left[-\frac{2}{\hbar} \text{Im} I\right] = \exp\left[-\frac{2}{\hbar} \text{Im} W_+\right], \quad (2.6)$$

$$\text{Prob[in]} \propto \exp\left[-\frac{2}{\hbar} \text{Im} I\right] = \exp\left[-\frac{2}{\hbar} \text{Im} W_-\right]. \quad (2.7)$$

Since  $W_+ = -W_-$ , we obtain the probability of a particle tunneling from inside to outside the horizon

$$\Gamma \propto \exp\left[-\frac{4}{\hbar} \text{Im} W_+\right]. \quad (2.8)$$

We take  $\hbar$  to be unity and also drop the '+' subscript from  $W$ . Integrating around the pole at the horizon leads to the result [15]

$$W = \frac{\pi \iota E}{\sqrt{f'(r_0)g'(r_0)}}. \quad (2.9)$$

Consequently, the tunneling probability becomes

$$\Gamma = \exp\left[-\frac{4\pi}{\sqrt{f'(r_0)g'(r_0)}} E\right] \quad (2.10)$$

which leads to the usual Hawking temperature

$$T_H = \frac{\sqrt{f'(r_0)g'(r_0)}}{4\pi}. \quad (2.11)$$

### 3 Accelerating and Rotating NUT Solutions

Black holes are the most important predictions of general relativity. The discovery of Schwarzschild BH is followed by the extension to the electrically charged version, Reissner–Nordström (RN) BH. This is followed by the discovery of rotating versions, Kerr and Kerr–Newman BHs [16]. The research in the BH area has further been extended by adding different sources like cosmological constant, an acceleration as well as a NUT parameter. Black hole solutions with these extensions belong to type-D class [17].

In general, the NUT parameter is associated with the gravitomagnetic monopole parameter of the central mass, or a twisting property of the surrounding spacetime but its exact physical meaning could not be ascertained. Recently, higher dimensional generalization of the Kerr–NUT–(anti) de Sitter spacetime [18] and its physical significance [19] is investigated. For this BH, the dominance of the NUT parameter over the rotation parameter leaves the spacetime free of curvature singularities and the corresponding solution is named as NUT-like solution. However, if the rotation parameter dominates the NUT parameter, the solution is Kerr-like and a ring curvature singularity forms. This kind of behavior on the singularity structure is independent of the presence of the cosmological constant.

There are many BH solutions which incorporate the NUT parameter and investigate its physical effect in the space of colliding waves. Exact interpretation of the NUT parameter becomes possible when a static Schwarzschild mass is immersed in a stationary, source free electromagnetic universe [20]. In this case, the NUT parameter is related to the twist of the electromagnetic universe excluding the central Schwarzschild mass. In the absence of this field, it reduces to the twist of the vacuum spacetime. Thus, the twist of the environmental space couples with the mass of the source to generate NUT parameter. It would be interesting to explore the tunneling phenomenon from such a BH.

The family of solutions which represents a pair of charged accelerating and rotating BHs with a nonzero NUT parameter can be written as [21]

$$\begin{aligned}
 ds^2 = & -\frac{1}{\Omega^2} \left\{ \frac{Q}{\rho^2} \left[ dt - \left( a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2} \right) d\phi \right]^2 - \frac{\rho^2}{Q} dr^2 \right. \\
 & \left. - \frac{\tilde{P}}{\rho^2} \left[ a dt - (r^2 + (a+l)^2) d\phi \right]^2 - \frac{\rho^2}{\tilde{P}} \sin^2 \theta d\theta^2 \right\}, \quad (3.1)
 \end{aligned}$$

where

$$\begin{aligned}
\Omega &= 1 - \frac{\alpha}{\omega}(l + a \cos \theta)r, \quad \rho^2 = r^2 + (l + a \cos \theta)^2, \\
Q &= \left[ (\omega^2 k + e^2 + g^2) \left( 1 + 2 \frac{\alpha l}{\omega} r \right) - 2Mr + \frac{\omega^2 k}{a^2 - l^2} r^2 \right] \\
&\quad \times \left[ 1 + \frac{\alpha(a-l)}{\omega} r \right] \left[ 1 - \frac{\alpha(a+l)}{\omega} r \right], \\
\tilde{P} &= \sin^2 \theta (1 - a_3 \cos \theta - a_4 \cos^2 \theta) = P \sin^2 \theta, \\
a_3 &= 2 \frac{\alpha a}{\omega} M - 4 \frac{\alpha^2 a l}{\omega^2} (\omega^2 k + e^2 + g^2), \quad a_4 = - \frac{\alpha^2 a^2}{\omega^2} (\omega^2 k + e^2 + g^2).
\end{aligned}$$

Here  $M$  represents a pair of BHs mass,  $e$  and  $g$  indicate electric and magnetic charges, respectively, while  $a$  shows a BH rotation and  $l$  is a NUT parameter [22]. The continuous parameter  $\alpha$  represents acceleration of the sources. Also, the rotation parameter  $\omega$  is related to the rotation of the sources and  $k$  is given by

$$\left( \frac{\omega^2}{a^2 - l^2} + 3\alpha^2 l^2 \right) k = 1 + 2 \frac{\alpha l}{\omega} M - 3 \frac{\alpha^2 l^2}{\omega^2} (e^2 + g^2), \quad (3.2)$$

where  $\alpha$ ,  $\omega$ ,  $M$ ,  $e$ ,  $g$  and  $k$  are arbitrary real parameters.

Notice that this type of BH involves acceleration  $\alpha$  while twisting behavior of the sources is proportional to the parameter  $\omega$  which is related to both the Kerr-like rotation parameter  $a$  and the NUT parameter  $l$ . Thus,  $\alpha$ ,  $M$ ,  $e$ ,  $g$ ,  $l$ ,  $a$  vary independently while  $\omega$  depends on nonzero value of rotation parameters  $l$  or  $a$ . For  $\alpha = 0$ , this reduces to the Kerr–Newman–NUT solution and Eq.(3.2) will become  $\omega^2 k = a^2 - l^2$ . In the absence of NUT parameter  $l = 0$ , it reduces to the pair of charged accelerating and rotating BHs. Further,  $\alpha = 0$  leads to the Kerr–Newman BH and  $a = 0$  yields the RN BH. In addition, if  $e = 0 = g$ , we have a Schwarzschild BH while  $l = 0 = a$  leads to the C-metric.

The metric (3.1) can also be written in a more suitable form as

$$ds^2 = -f(r, \theta) dt^2 + \frac{dr^2}{g(r, \theta)} + \Sigma(r, \theta) d\theta^2 + K(r, \theta) d\phi^2 - 2H(r, \theta) dt d\phi, \quad (3.3)$$

where  $f(r, \theta)$ ,  $g(r, \theta)$ ,  $\Sigma(r, \theta)$ ,  $K(r, \theta)$  and  $H(r, \theta)$  can be defined as follows:

$$f(r, \theta) = \left( \frac{Q - Pa^2 \sin^2 \theta}{\rho^2 \Omega^2} \right), \quad g(r, \theta) = \frac{Q \Omega^2}{\rho^2}, \quad \Sigma(r, \theta) = \frac{\rho^2}{\Omega^2 P}, \quad (3.4)$$

$$K(r, \theta) = \frac{1}{\rho^2 \Omega^2} \left[ \sin^2 \theta P [r^2 + (a + l)^2]^2 - Q (a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2})^2 \right] \quad (3.5)$$

$$H(r, \theta) = \frac{1}{\rho^2 \Omega^2} \left[ \sin^2 \theta P a [r^2 + (a + l)^2] - Q (a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2}) \right]. \quad (3.6)$$

The electromagnetic vector potential for these BHs is given by [23]

$$A = \frac{1}{a[r^2 + (l + a \cos \theta)^2]} [-er[adt - d\phi\{(l + a)^2 - (l^2 + a^2 \cos^2 \theta + 2la \cos \theta)\}] - g(l + a \cos \theta)[adt - d\phi\{r^2 + (l + a)^2\}]]. \quad (3.7)$$

The event horizons are obtained for  $g(r, \theta) = \frac{\Delta(r)}{\Sigma(r, \theta)} = 0$  [11], where  $\Delta(r) = \frac{Q}{P}$ . This implies that  $\Delta(r) = 0 = Q$ , yielding the horizon radii

$$r_{\alpha_1} = \frac{\omega}{\alpha(a + l)}, \quad r_{\alpha_2} = -\frac{\omega}{\alpha(a - l)}, \quad r_{\pm} = \frac{a^2 - l^2}{\omega^2 k} \left[ -[(\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - M] \pm \sqrt{[(\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - M]^2 - \frac{\omega^2 k}{a^2 - l^2} (\omega^2 k + e^2 + g^2)} \right] \quad (3.8)$$

where  $r_{\pm}$  represent the outer and inner horizons, respectively, such that  $[(\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - M]^2 - \frac{\omega^2 k}{a^2 - l^2} (\omega^2 k + e^2 + g^2) > 0$ ,  $r_{\alpha_1}$  and  $r_{\alpha_2}$  are acceleration horizons. The angular velocity at the BH horizon can be defined as

$$\Omega_H = \frac{H(r_+, \theta)}{K(r_+, \theta)} = \frac{a}{r_+^2 + (a + l)^2}. \quad (3.9)$$

The inverse function of  $f(r, \theta)$  is

$$F(r, \theta) = f(r, \theta) + \frac{H^2(r, \theta)}{K(r, \theta)}. \quad (3.10)$$

For these BHs, this can be written in the following form:

$$F(r, \theta) = \frac{PQ \sin^2 \theta \rho^2}{\Omega^2 [\sin^2 \theta P [r^2 + (a + l)^2]^2 - Q (a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2})^2]}. \quad (3.11)$$

In terms of  $\Delta(r)$  and  $\Sigma$ , we can write the inverse function of  $f(r, \theta)$  as

$$F(r, \theta) = \frac{P^2 \Delta(r) \Sigma(r, \theta)}{[r^2 + (a + l)^2]^2 - \Delta(r) \sin^2 \theta [a + \frac{2l}{1 + \cos \theta}]^2}. \quad (3.12)$$

## 4 Tunneling of Charged Fermions

In order to study charged fermions tunneling of mass  $m$  from a pair of accelerating and rotating BHs with NUT parameter having electric and magnetic charges, the covariant Dirac equation with electric charge  $q$  is given by [24]

$$\iota\gamma^\mu \left( D_\mu - \frac{\iota q}{\hbar} A_\mu \right) \Psi + \frac{m}{\hbar} \Psi = 0, \quad \mu = 0, 1, 2, 3 \quad (4.1)$$

where  $A_\mu$  is the 4-potential,  $\Psi$  is the wave function and

$$D_\mu = \partial_\mu + \Omega_\mu, \quad \Omega_\mu = \frac{1}{2} \iota \Gamma_\mu^{\alpha\beta} \Sigma_{\alpha\beta}, \quad \Sigma_{\alpha\beta} = \frac{1}{4} \iota [\gamma^\alpha, \gamma^\beta]. \quad (4.2)$$

The antisymmetric property of the Dirac matrices [12], i.e.,  $[\gamma^\alpha, \gamma^\beta] = 0$  for  $\alpha = \beta$  and  $[\gamma^\alpha, \gamma^\beta] = -[\gamma^\beta, \gamma^\alpha]$  for  $\alpha \neq \beta$ , reduces the Dirac equation (4.1) in the following form:

$$\iota\gamma^\mu \left( \partial_\mu - \frac{\iota q}{\hbar} A_\mu \right) \Psi + \frac{m}{\hbar} \Psi = 0. \quad (4.3)$$

The Dirac matrices are given by

$$\begin{aligned} \gamma^t &= \frac{1}{\sqrt{F(r, \theta)}} \begin{pmatrix} \iota & 0 \\ 0 & -\iota \end{pmatrix}, \quad \gamma^r = \sqrt{g(r, \theta)} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \\ \gamma^\theta &= \frac{1}{\sqrt{\Sigma(r, \theta)}} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \\ \gamma^\phi &= \frac{1}{\sqrt{K(r, \theta)}} \left[ \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} + \frac{H(r, \theta)}{\sqrt{F(r, \theta)K(r, \theta)}} \begin{pmatrix} \iota & 0 \\ 0 & -\iota \end{pmatrix} \right], \end{aligned}$$

where  $\sigma^i$  ( $i = 1, 2, 3$ ) are the Pauli sigma matrices defined as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -\iota \\ \iota & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (4.4)$$

The spinor wave function  $\Psi$  (related to the particle's action) has two spin states: spin-up (in +ve  $r$ -direction) and spin-down (in -ve  $r$ -direction). For



the spin-up and spin-down particle's solution, we assume [10]

$$\Psi_{\uparrow}(t, r, \theta, \phi) = \begin{bmatrix} A(t, r, \theta, \phi) \\ 0 \\ B(t, r, \theta, \phi) \\ 0 \end{bmatrix} \exp \left[ \frac{\iota}{\hbar} I_{\uparrow}(t, r, \theta, \phi) \right], \quad (4.5)$$

$$\Psi_{\downarrow}(t, r, \theta, \phi) = \begin{bmatrix} 0 \\ C(t, r, \theta, \phi) \\ 0 \\ D(t, r, \theta, \phi) \end{bmatrix} \exp \left[ \frac{\iota}{\hbar} I_{\downarrow}(t, r, \theta, \phi) \right], \quad (4.6)$$

where  $I_{\uparrow/\downarrow}$  denote the emitted spin-up/spin-down particle's action, respectively. Note that we shall only analyze the spin-up case as the spin-down case is just analogous.

The particle's action is given by the following ansatz (2.4):

$$I_{\uparrow} = -Et + J\phi + W(r, \theta), \quad (4.7)$$

where  $E$ ,  $J$  and  $W$  are energy, angular momentum and arbitrary function, respectively. Using this ansatz into the Dirac equation with  $\iota A = B$ ,  $\iota B = A$  and Taylor's expansion of  $F(r, \theta)$  near the event horizon, it follows that

$$- B \left[ \frac{-E + \Omega_H J + \frac{qer}{[r_+^2 + (a+l)^2]}}{\sqrt{(r-r_+)\partial_r F(r_+, \theta)}} + \sqrt{(r-r_+)\partial_r g(r_+, \theta)}(\partial_r W) \right] + mA = 0, \quad (4.8)$$

$$\begin{aligned} & - B \left[ \sqrt{\frac{P\Omega^2(r_+, \theta)}{\rho^2(r_+, \theta)}}(\partial_{\theta} W) \right. \\ & + \frac{\iota\rho(r_+, \theta)\Omega(r_+, \theta)}{\sqrt{\sin^2 \theta P[r^2 + (a+l)^2]^2 - Q(a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2})^2}} \\ & \times \left\{ J - q \left[ \frac{er[(l+a)^2 - (l^2 + a^2 \cos^2 \theta + 2la \cos \theta)]}{a[r^2 + (l+a \cos \theta)^2]} \right] \right. \\ & \left. \left. + \frac{g(l+a \cos \theta)[r^2 + (l+a)^2]}{a[r^2 + (l+a \cos \theta)^2]} \right] \right\} \Big] = 0, \quad (4.9) \end{aligned}$$

$$A \left[ \frac{-E + \Omega_H J + \frac{qer}{[r_+^2 + (a+l)^2]}}{\sqrt{(r-r_+)\partial_r F(r_+, \theta)}} - \sqrt{(r-r_+)\partial_r g(r_+, \theta)}(\partial_r W) \right] + mB = 0, \quad (4.10)$$

$$\begin{aligned} & - A \left[ \sqrt{\frac{P\Omega^2(r_+, \theta)}{\rho^2(r_+, \theta)}}(\partial_\theta W) \right. \\ & + \frac{\iota\rho(r_+, \theta)\Omega(r_+, \theta)}{\sqrt{\sin^2 \theta P[r^2 + (a+l)^2] - Q(a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2})^2}} \\ & \times \left\{ J - q \left[ \frac{er[(l+a)^2 - (l^2 + a^2 \cos^2 \theta + 2la \cos \theta)]}{a[r^2 + (l+a \cos \theta)^2]} \right. \right. \\ & \left. \left. + \frac{g(l+a \cos \theta)[r^2 + (l+a)^2]}{a[r^2 + (l+a \cos \theta)^2]} \right] \right\} \Big] = 0. \end{aligned} \quad (4.11)$$

The arbitrary function  $W(r, \theta)$  can be separated as follows: [11]

$$W(r, \theta) = R(r) + \Theta(\theta). \quad (4.12)$$

First we solve Eqs.(4.8)–(4.11) for the massless case ( $m = 0$ ). Using the above separation, Eqs.(4.8) and (4.10) reduce to

$$-B \left[ \frac{-E + \Omega_H J + \frac{qer}{[r_+^2 + (a+l)^2]}}{\sqrt{(r-r_+)\partial_r F(r_+, \theta)}} + \sqrt{(r-r_+)\partial_r g(r_+, \theta)}R'(r) \right] = 0, \quad (4.13)$$

$$A \left[ \frac{-E + \Omega_H J + \frac{qer}{[r_+^2 + (a+l)^2]}}{\sqrt{(r-r_+)\partial_r F(r_+, \theta)}} - \sqrt{(r-r_+)\partial_r g(r_+, \theta)}R'(r) \right] = 0, \quad (4.14)$$

implying that

$$\begin{aligned} R'(r) = R'_+(r) &= -R'_-(r) = \left[ \frac{[r_+^2 + (a+l)^2]}{(r-r_+)[1 + \frac{\alpha(a-l)}{\omega}r_+][1 - \frac{\alpha(a+l)}{\omega}r_+]} \right. \\ &\times \left. \frac{[E - \Omega_H J - \frac{qer_+}{[r_+^2 + (a+l)^2]}]}{2[\frac{\alpha l}{\omega}(\omega^2 k + e^2 + g^2) - M + \frac{\omega^2 k}{a^2 - l^2}r_+]} \right], \end{aligned} \quad (4.15)$$

where  $R_+$  and  $R_-$  correspond to the outgoing and incoming solutions, respectively. This equation represents the pole at the horizon,  $r = r_+$ .

In order to calculate the Hawking temperature by using the tunneling approach, we need to regularize the singularity by specifying a suitable complex contour to bypass the pole. In standard coordinate representation, for

outgoing particles (from inside of the horizon to outside), we should take the contour to be an infinitesimal semicircle below the pole  $r = r_+$ . Similarly, for the ingoing particles (from outside to inside), the contour is above the pole. Here, for the purpose of calculating the semiclassical tunneling probability, we need to multiply the resulting wave equation by its complex conjugate. In this way, the part of trajectory that starts from outside of the BH and continues to the observer, will not contribute to the calculation of the final tunneling probability and can be ignored (since it will be entirely real). Therefore, the only part of the wave equation (trajectory) that contributes to the tunneling probability is the contour around the BH horizon. Instead we choose a mathematically equivalent convention that the outgoing contour is in the lower half plane and so do not multiply by a negative sign [11].

Integrating Eq.(4.15) around the pole, we obtain

$$R_+(r) = -R_-(r) = \left[ \frac{\pi l [r_+^2 + (a+l)^2]}{[1 + \frac{\alpha(a-l)}{\omega} r_+][1 - \frac{\alpha(a+l)}{\omega} r_+]} \frac{[E - \Omega_H J - \frac{qer_+}{r_+^2 + (a+l)^2}]}{2[\frac{\alpha l}{\omega}(\omega^2 k + e^2 + g^2) - M + \frac{\omega^2 k}{a^2 - l^2} r_+]} \right]. \quad (4.16)$$

The imaginary parts of  $R_+$  and  $R_-$  yield

$$\text{Im}R_+ = -\text{Im}R_- = \left[ \frac{\pi [r_+^2 + (a+l)^2]}{[1 + \frac{\alpha(a-l)}{\omega} r_+][1 - \frac{\alpha(a+l)}{\omega} r_+]} \frac{[E - \Omega_H J - \frac{qer_+}{r_+^2 + (a+l)^2}]}{2[\frac{\alpha l}{\omega}(\omega^2 k + e^2 + g^2) - M + \frac{\omega^2 k}{a^2 - l^2} r_+]} \right]. \quad (4.17)$$

Thus the particle's tunneling probability from inside to outside the horizon is

$$\begin{aligned} \Gamma = \frac{\text{Prob[out]}}{\text{Prob[in]}} &= \frac{\exp[-2(\text{Im}R_+ + \text{Im}\Theta)]}{\exp[-2(\text{Im}R_- + \text{Im}\Theta)]} = \exp[-4\text{Im}R_+] \\ &= \exp \left[ \frac{-2\pi [r_+^2 + (a+l)^2]}{[1 + \frac{\alpha(a-l)}{\omega} r_+][1 - \frac{\alpha(a+l)}{\omega} r_+]} \frac{[E - \Omega_H J - \frac{qer_+}{r_+^2 + (a+l)^2}]}{[\frac{\alpha l}{\omega}(\omega^2 k + e^2 + g^2) - M + \frac{\omega^2 k}{a^2 - l^2} r_+]} \right]. \end{aligned} \quad (4.18)$$

Here,  $\Gamma$  is the tunneling probability for the classically forbidden trajectories of the s-waves coming from inside to outside the horizon. Using the WKB approximation,  $\Gamma$  is given by Eq.(1.1) in terms of classical action  $I$  of Dirac particles tunneling across the BH horizon as trajectories up to leading order in  $\hbar$ . Thus, for calculating the Hawking temperature, we expand the action in terms of particles energy  $E$ , i.e.,  $2I = \beta E + O(E^2)$  so that the Hawking temperature is recovered at linear order given by

$$\Gamma \sim \exp[-2I] \simeq \exp[-\beta E]. \quad (4.19)$$

This shows that the emission rate in the tunneling approach, up to first order in  $E$ , retrieves the Boltzmann factor,  $\exp[-\beta E]$ , where  $\beta = \frac{1}{T_H}$  [15]. The higher-order terms represent the self-interaction effects resulting from the energy conservation.

Semiclassically, for the charged rotating BH solution, the energy  $E$  of the emitted particle in the Boltzmann factor should be replaced by  $E - \Omega_H J$  due to the presence of the ergosphere. The corresponding Hawking temperature at the event horizon takes the form

$$T_H = \left[ \frac{[\frac{\alpha l}{\omega}(\omega^2 k + e^2 + g^2) - M + \frac{\omega^2 k}{a^2 - l^2} r_+][1 + \frac{\alpha(a-l)}{\omega} r_+][1 - \frac{\alpha(a+l)}{\omega} r_+]}{2\pi[r_+^2 + (a+l)^2]} \right]. \quad (4.20)$$

Thus, the Hawking temperature of Dirac particles tunneling from the event horizon of the charged accelerating and rotating BHs with NUT parameter is well described via fermions tunneling method. When  $l = 0$ ,  $k = 1$  in Eq.(4.20), the Hawking temperature of the accelerating and rotating BHs with electric and magnetic charges is recovered [12]. When  $\alpha = 0$ , the Hawking temperature of the charged accelerating and rotating BHs is reduced to the temperature of non-accelerating BHs [13]. For  $l = 0$ ,  $k = 1$ ,  $\alpha = 0$  in Eq.(4.20), the Hawking temperature of the Kerr–Newman BH [11] is obtained which is further reduced to the temperature of the RN BH (for  $a = 0$ ). Finally, in the absence of charge, the temperature exactly reduces to the Hawking temperature of the Schwarzschild BH [25].

For the massive case ( $m \neq 0$ ), following the same steps, we can obtain the same temperature (4.20). Thus the behavior of massive particles near the BH horizon is the same as that for the massless particles. For  $l = 0$ , the tunneling probability (4.18) reduces to the form for the pair of accelerating and rotating BHs [12]. For  $\alpha = 0$ , we recover the tunneling probability for

the Kerr–Newman BH [11]. In the absence of rotation, this yields the same result as for the RN BH [24].

Now we explore the tunneling probability of charged massive and massless fermions from the acceleration horizon  $r_{\alpha_1}$  given in Eq.(3.8). The corresponding set of Eqs.(4.8)–(4.11) for the outgoing and incoming fermions, respectively, yield

$$R_+(r) = -R_-(r) = \left[ \frac{\pi \iota [r_{\alpha_1}^2 + (a+l)^2]}{\frac{\alpha}{\omega} [l + \frac{\alpha}{\omega} (a^2 - l^2) r_{\alpha_1}]} \frac{[E - \Omega_\alpha J - \frac{qer_{\alpha_1}}{[r_{\alpha_1}^2 + (a+l)^2]}]}{2[(\omega^2 k + e^2 + g^2)(1 + \frac{2\alpha l r_{\alpha_1}}{\omega}) - 2Mr_{\alpha_1} + \frac{\omega^2 k r_{\alpha_1}^2}{a^2 - l^2}]} \right]. \quad (4.21)$$

The tunneling probability can be written as

$$\Gamma = \exp \left[ \frac{-2\pi [r_{\alpha_1}^2 + (a+l)^2]}{\frac{\alpha}{\omega} [l + \frac{\alpha}{\omega} (a^2 - l^2) r_{\alpha_1}]} \frac{[E - \Omega_\alpha J - \frac{qer_{\alpha_1}}{[r_{\alpha_1}^2 + (a+l)^2]}]}{[(\omega^2 k + e^2 + g^2)(1 + \frac{2\alpha l r_{\alpha_1}}{\omega}) - 2Mr_{\alpha_1} + \frac{\omega^2 k r_{\alpha_1}^2}{a^2 - l^2}]} \right]. \quad (4.22)$$

Consequently, the Hawking temperature for the acceleration horizon turns out to be

$$T_H = \left[ \frac{\frac{\alpha}{\omega} [(\omega^2 k + e^2 + g^2)(1 + \frac{2\alpha l r_{\alpha_1}}{\omega}) - 2Mr_{\alpha_1} + \frac{\omega^2 k r_{\alpha_1}^2}{a^2 - l^2}] [l + \frac{\alpha}{\omega} (a^2 - l^2) r_{\alpha_1}]}{2\pi [r_{\alpha_1}^2 + (a+l)^2]} \right]. \quad (4.23)$$

## 5 Action for the Emitted Particles

In order to obtain the explicit expression for the action  $I_\uparrow$  in the spin-up case, we solve Eqs.(4.8)–(4.11) near the BH horizon. Using Eq.(4.12), we can write Eq.(4.8) in the form

$$R'(r) = \frac{mA}{B\sqrt{(r-r_+)\partial_r g(r_+, \theta)}} - \frac{-E + \Omega_H J + \frac{qer_+}{[r_+^2 + (a+l)^2]}}{(r-r_+)\sqrt{\partial_r F(r_+, \theta)\partial_r g(r_+, \theta)}}. \quad (5.1)$$

Integrating with respect to  $r$ , we get

$$R(r) = R_+(r) = \int \frac{mA}{B\sqrt{(r-r_+)\partial_r g(r_+, \theta)}} dr - \frac{\left(-E + \Omega_H J + \frac{qer_+}{[r_+^2 + (a+l)^2]}\right)}{\sqrt{\partial_r F(r_+, \theta)\partial_r g(r_+, \theta)}} \ln(r-r_+). \quad (5.2)$$

Similarly, for the incoming particles, Eq.(4.10) yields

$$R(r) = R_-(r) = \int \frac{mB}{A\sqrt{(r-r_+)\partial_r g(r_+, \theta)}} dr + \frac{\left(-E + \Omega_H J + \frac{qer_+}{[r_+^2 + (a+l)^2]}\right)}{\sqrt{\partial_r F(r_+, \theta)\partial_r g(r_+, \theta)}} \ln(r-r_+). \quad (5.3)$$

Using Eq.(4.12), Eqs.(4.9) and (4.11) imply that

$$\begin{aligned} & \sqrt{\frac{P\Omega^2(r_+, \theta)}{\rho^2(r_+, \theta)}} \partial_\theta \Theta + \frac{\iota\rho(r_+, \theta)\Omega(r_+, \theta)}{\sqrt{\sin^2 \theta P[r_+^2 + (a+l)^2]^2}} \\ & \times \left[ J - q \left\{ \frac{er_+[(l+a)^2 - (l^2 + a^2 \cos^2 \theta + 2la \cos \theta)]}{a[r_+^2 + (l+a \cos \theta)^2]} \right. \right. \\ & \left. \left. + \frac{g(l+a \cos \theta)[r_+^2 + (l+a)^2]}{a[r_+^2 + (l+a \cos \theta)^2]} \right\} \right] = 0. \end{aligned} \quad (5.4)$$

Substituting the values of  $\rho$  and  $P$ , after some manipulation, it follows

that

$$\begin{aligned}
\partial_\theta \Theta &= \frac{\iota a^2 \sin \theta J + \iota q e r_+ a \sin \theta}{[r_+^2 + (a+l)^2]} \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} - \cos^2 \theta \left\{ -\alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1} \\
&\quad + \frac{-\iota a J + \iota q g (l + a \cos \theta)}{a \sin \theta} \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} - \cos^2 \theta \left\{ -\alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1} \\
&\quad + \frac{2l a \iota (1 - \cos \theta) J + \iota q e r_+ 2l (1 - \cos \theta)}{\sin \theta [r_+^2 + (a+l)^2]} \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} - \cos^2 \theta \left\{ -\alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1}.
\end{aligned} \tag{5.5}$$

Integrating with respect to  $\theta$ , we have

$$\Theta = \frac{\iota [q a e r_+ + J a^2]}{[r_+^2 + (a+l)^2]} I_1 + I_2 + \frac{2l \iota [a J + q e r_+]}{[r_+^2 + (a+l)^2]} I_3, \tag{5.6}$$

where  $I_1$ ,  $I_2$  and  $I_3$  are given as follows

$$\begin{aligned}
I_1 &= \int \sin \theta \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right. \\
&\quad \left. - \cos^2 \theta \left\{ -\alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1} d\theta,
\end{aligned} \tag{5.7}$$

$$\begin{aligned}
I_2 &= \int \left[ \frac{\iota \{q g (l + a \cos \theta) - a J\}}{a \sin \theta} \right] \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} - \cos^2 \theta \left\{ -\alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1} d\theta,
\end{aligned} \tag{5.8}$$

$$\begin{aligned}
I_3 &= \int \left[ \frac{1 - \cos \theta}{\sin \theta} \right] \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right. \\
&\quad \left. - \cos^2 \theta \left\{ -\alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1} d\theta.
\end{aligned} \tag{5.9}$$

Solving these integrals, we obtain after some algebra

$$\begin{aligned}
I_1 = & \left[ \frac{1}{2\alpha\frac{a}{\omega}\sqrt{\left\{M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2)\right\}^2 - (\sqrt{\omega^2k + e^2 + g^2})^2}} \right] \\
& \times \ln \left[ \left( 1 - \alpha\frac{a}{\omega} \cos \theta \left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right. \right. \right. \\
& + \left. \left. \left. \sqrt{\left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2k + e^2 + g^2})^2} \right\} \right) \right. \\
& \times \left( 1 - \alpha\frac{a}{\omega} \cos \theta \left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right. \right. \\
& - \left. \left. \left. \sqrt{\left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2k + e^2 + g^2})^2} \right\} \right) \right]^{-1}, \tag{5.10}
\end{aligned}$$

$$\begin{aligned}
I_2 = & L_1 \ln \left[ \left( 1 - \alpha\frac{a}{\omega} \cos \theta \left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right. \right. \right. \\
& + \left. \left. \left. \sqrt{\left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2k + e^2 + g^2})^2} \right\} \right) \right. \\
& \times \left( 1 - \alpha\frac{a}{\omega} \cos \theta \left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right. \right. \\
& - \left. \left. \left. \sqrt{\left\{ M - 2\alpha\frac{l}{\omega}(\omega^2k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2k + e^2 + g^2})^2} \right\} \right) \right]^{-1} \\
& + L_2 \ln \left[ 1 - 2 \cos \theta \left\{ \alpha\frac{a}{\omega}M - 2\alpha^2l\frac{a}{\omega^2}(\omega^2k + e^2 + g^2) \right\} \right. \\
& + \left. \alpha^2\frac{a^2}{\omega^2} \cos^2 \theta (\omega^2k + e^2 + g^2) \right] + L_3 \ln[1 - \cos \theta] \\
& + L_4 \ln[1 + \cos \theta], \tag{5.11}
\end{aligned}$$



where

$$\begin{aligned}
L_1 = & \left[ \left\{ 2\alpha \frac{a}{\omega} \sqrt{\left\{ M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2 k + e^2 + g^2})^2} \right\} \right. \\
& \times \left\{ \left( 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right)^2 - 4\alpha^2 \frac{a^2}{\omega^2} \right. \\
& \times \left. \left( M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right)^2 \right\} \Big]^{-1} \left[ \iota J \left\{ 2 \left( \alpha \frac{a}{\omega} M \right. \right. \right. \\
& - \left. 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right)^2 - \alpha^4 \frac{a^4}{\omega^4} (\omega^2 k + e^2 + g^2)^2 \\
& - \left. \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} + \iota q g \left\{ -\alpha \frac{a}{\omega} M + 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right. \\
& + \left. \frac{\alpha^4 l}{a} \frac{a^4}{\omega^4} (\omega^2 k + e^2 + g^2)^2 - \frac{2\alpha^2 l}{a} \frac{a^2}{\omega^2} \left( M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right)^2 \right. \\
& + \left. \frac{\alpha^2 l}{a} \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) + \alpha^3 \frac{a^3}{\omega^3} (\omega^2 k + e^2 + g^2) \right. \\
& \times \left. \left. \left. \left( M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right) \right\} \right] \right], \tag{5.12}
\end{aligned}$$

$$\begin{aligned}
L_2 = & \left[ \left\{ 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\}^2 - 4\alpha^2 \frac{a^2}{\omega^2} \right. \\
& \times \left. \left\{ M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right\}^2 \right]^{-1} \\
& \times \left[ \iota J \left\{ \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right. \\
& - \frac{\iota q g}{2} \left\{ 2 \frac{l}{a} \left( \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right) \right. \\
& + \left. \left. 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right], \tag{5.13}
\end{aligned}$$

$$\begin{aligned}
L_3 = & \frac{1}{2} \left[ \iota q g \left( \frac{l}{a} + 1 \right) - \iota J \right] \left[ 1 - 2 \left\{ \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right. \\
& \times \left. \left. \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right]^{-1}, \tag{5.14}
\end{aligned}$$

$$\begin{aligned}
L_4 = & \frac{1}{2} \left[ \iota J + \iota q g \left( 1 - \frac{l}{a} \right) \right] \left[ 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \right. \\
& + \left. 2 \left\{ \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1} \tag{5.15}
\end{aligned}$$

and  $I_3$  can be obtained as

$$\begin{aligned}
I_3 = & N_1 \ln \left[ \left( 1 - \alpha \frac{a}{\omega} \cos \theta \left\{ M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right. \right. \right. \\
& + \left. \left. \left. \sqrt{\left\{ M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2 k + e^2 + g^2})^2} \right\} \right) \right. \\
& \times \left( 1 - \alpha \frac{a}{\omega} \cos \theta \left\{ M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right. \right. \\
& - \left. \left. \left. \sqrt{\left\{ M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2) \right\}^2 - (\sqrt{\omega^2 k + e^2 + g^2})^2} \right\} \right)^{-1} \right] \\
& + N_2 \ln \left[ 1 - 2 \cos \theta \left\{ \alpha \frac{a}{\omega} M - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right. \\
& + \left. \left. \alpha^2 \frac{a^2}{\omega^2} \cos^2 \theta (\omega^2 k + e^2 + g^2) \right] + N_3 \ln[1 + \cos \theta], \tag{5.16}
\end{aligned}$$

where

$$\begin{aligned}
N_1 &= \left[ \frac{\alpha \frac{a}{\omega} M - 2l\alpha^2 \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2)}{2\alpha \frac{a}{\omega} \sqrt{\{M - 2\alpha \frac{l}{\omega} (\omega^2 k + e^2 + g^2)\}^2 - (\sqrt{\omega^2 k + e^2 + g^2})^2}} \right] \\
&\times \left[ 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) + 2\alpha \left\{ \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1}, \tag{5.17}
\end{aligned}$$

$$\begin{aligned}
N_2 &= \frac{1}{2} \left[ 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) + 2 \left\{ \alpha \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1}, \tag{5.18}
\end{aligned}$$

$$\begin{aligned}
N_3 &= - \left[ 1 + \alpha^2 \frac{a^2}{\omega^2} (\omega^2 k + e^2 + g^2) + 2 \left\{ \alpha \frac{a}{\omega} M \right. \right. \\
&\quad \left. \left. - 2\alpha^2 l \frac{a}{\omega^2} (\omega^2 k + e^2 + g^2) \right\} \right]^{-1}. \tag{5.19}
\end{aligned}$$

Inserting these values of the integrals in Eq.(5.6) and using Eq.(4.12), we obtain  $W(r, \theta)$ . This leads one to evaluate the action for the outgoing massive particles. For  $m = 0$ , this reduces to the action for the massless particles. Similarly, we can determine the action for the incoming massive and massless particles.

## 6 Outlook

In this paper, we have used semiclassical WKB approximation to study tunneling of charged fermions from a pair of accelerating and rotating BHs having electric and magnetic charges, together with a NUT parameter. Classically, a particle can only fall inside the horizon while in the semiclassical approach, the horizon plays a role of two way energy barrier for a pair of positive and negative energy particles. The positive energy particles have ability to tunnel outside the event horizon, which contradicts classical approach. Thus, we have considered tunneling probabilities for both incoming as well as outgoing particles. Relating these tunneling probabilities with the Boltzmann factor  $\exp[-\beta E]$  for emission at the Hawking temperature, we can recover the corresponding Hawking temperature for this pair of BHs at event horizons.

The tunneling probabilities of outgoing/incoming charged fermions do not depend upon mass of the fermions but only its charge. The corresponding Hawking temperature depends upon mass, acceleration and rotation parameters and also NUT parameter as well as electric and magnetic charges of the pair of BHs. Equations for the spin-down case are of the same form as for the spin-up case except for a negative sign. For both massive and massless cases, the Hawking temperature implies that both spin-up and spin-down particles are emitted at the same rate [10].

It is worth mentioning here that in the absence of the NUT parameter  $l = 0$ , all the results reduce to the results of the pair of charged accelerating and rotating BHs [12]. Further,  $\alpha = 0$  provides the results of the Kerr–Newman BH [11] and  $a = 0$  yields the results of the RN BH [24].

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